Advanced Probability (106349) Spring 2005

Instructor: Professor Ross Pinsky (pinsky@math.technion.ac.il)

Time: Monday: 1:30-2:30, Thursday: 10:30-12:30

Prerequisites:

1. First course in probability—for example, *Probability Theory* (104222)

2. Basic knowledge of measure theory—for example, the material in *Real Functions* (104165).

Course outline

1. Introduction: Measure-theoretic underpinnings of probability: random variables, expected value, independence. Dynkin's π-λ theorem.

2. Convergence theorems for sequences of independent random variables: weak law of large numbers, the lemma of Borel-Cantelli, convergence of random series—Kolmogorov three series theorem, strong law of large numbers, examples.

3. Weak convergence of probability measures on the real line—Helly's selection theorem.

Weak convergence of probability measures in Polish spaces: tightness criterion, examples.

4. Characteristic functions and tightness—the continuity theorem.

The Lindeberg-Feller central limit theorem.

5. Markov chains—finite and countably infinite: asymptotic stationarity, transient and recurrent states, examples: birth and death processes, random walks on R^n and on finite groups.

6. (If there is time) Basic ergodic theory.

Reserved Books

1. Richard Durrett: *Probability: Theory and Examples*

2. K. L. Chung: *A Course in Probability Theory*

3. L. Breiman: *Probability*

4. P. Billingsley: *Probability and Measure*

5. E. Cinlar: *Introduction to Stochastic Processes*

6. B. Fristedt and L. Gray: *A Modern Approach to Probability Theory*

7. S.R.S. Varadhan: *Stochastic Processes*