**Stochastic Processes 106429**

*Lecturer:* Professor Ross Pinsky

*Time:* Monday 9:30-10:30, Wednesday 9:30-11:30.

*Prerequisite:* Probability Theory (104222), Advanced Probability (106349). Some background in measure theory is essential! The course Real Functions (104165) is sufficient.

*Course Outline*

**I. Discrete Time Martingales:** conditional expectation (via the Radon-Nikodym derivative), definition of a martingale, sub/super martingale, martingale convergence theorem, Doob's inequality, $L^p$ maximum inequality, uniform integrability, stopping times and Doob's optional sampling theorem, applications of martingale theory.

**II. continuous Time Stochastic Processes:** definitions, separability, stochastic continuity, Kolomogorov's theorem for almost sure continuity, continuous time versions of the discrete time martingale theory.

**III. Brownian Motion and its Properties:** definition and several alternative constructions, continuity, Brownian scaling, quadratic variation, almost sure nowhere differentiability, law of the iterated logarithm, strong Markov property, the zeros of Brownian motion, first exit distributions.

**IV. Stochastic Integrals:** construction, Itô's formula.

**V. Stochastic Differential Equations** (Time permitting).

**Recommended Books**

**1.** *Probability: Theory and Examples* by Richard Durrett

**2.** *Probability* by Leo Breiman

**3.** *Brownian Motion and Stochastic Calculus* by Ioannis Karatzas and Steven Shreve

**4.** *Stochastic Differential Equations and Applications, Vol. 1* by Avner Friedman

**5.** *Introduction to Stochastic Integration* by K. L. Chung and Ruth Williams