Long Range Dependence and Heavy Tails

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Time: Mondays, 14:30-16:30. The course starts on 26/10 (not 19/10).
Place: Meyer (EE) Building, Room 916 (changed).

Description: Given a stochastic process $(X(t),\, t \geq 0)$, long range dependence (sometimes called long term memory) refers to non-negligible dependence between the current value $X(t)$ and the initial value $X(0)$, which persists even as t increases. The history of long range dependence started with Hurst's classical work in the 1950's regarding a dam design project in the Nile River. Since then, long range dependence has traditionally been defined by second order properties of stochastic processes, classifying long/short memory of the process by the asymptotic behavior of covariances. Such a classical definition becomes ambiguous when applied to heavy tailed processes. In particular, using the second order property to distinguish between long/short range dependence is nonsensical if the process has infinite moments, which makes the theory of long memory processes with heavy tailed marginals an exciting and ongoing research area.

This course covers a wide range of topics regarding long range dependence, including the historical background, a classical treatment based on second order properties, as well as connections with ergodic theory, self-similarity, heavy tailed processes, and various limit theorems.

The specific topics to be treated will include:

- Some history. The Hurst phenomenon.
  - R/S statistics.
  - Fractional Brownian motion.
- Long term memory, Ergodic theory, Strong mixing.
- Second-order theory.
  - Spectral representation of stationary processes.
  - Hermite polynomials.
- Self-similar processes.
  - Lamperti's theorem.
  - Self-similar processes with stationary increments.

  - Linear fractional processes.

- Limit theorems for long term memory, heavy tailed processes.
  - Heavy tails plus dependence.
  - General tools: regular variation, stable laws, point processes.
  - Heavy tailed moving average processes.
  - Long memory stochastic volatility models.
- Long range dependence and flow properties (if time allows).
  - Infinite ergodic theory.
  - Infinitely divisible processes.
  - Conservative flow and dissipative flow.

Prerequisites: A basic knowledge about measure theoretic probability is assumed.

Textbook: We shall mostly follow the textbook

G. Samorodnitsky (2007), *Long Range Dependence*, volume 1:3 of *Foundations and Trends in Stochastic Systems*,

but more details of the proofs and backgrounds will be provided. Some other topics, including recent results, which are not given in the above textbook, will also be covered. The following are useful references as well.

J. Beran et. al (2013), *Long-Memory Processes: Probabilistic Properties and Statistical Methods*, Springer.

W. Whitt (2002), *Stochastic-Process Limits. An Introduction to Stochastic-Process Limits and Their Applications to Queues*, Springer.