

Following is the plan of a course by Haya Kaspi. The course will be given in the spring semester of 2015 on Thursdays 13:30-15:30 in room 527(seminar room) of the Bloomfield building.

Levy Processes, Excursion Theory and Local Times of Markov Processes

(1) Introduction.

- Infinitely divisible distributions and the Levy Khinchine formula.
- Martingales
- Poisson Processes and Poisson point processes
- Compensated sums of jumps.

(2) Levy processes as Markov processes

- Levy processes and the Levy Khinchine formula.
- The Markov and Strong Markov properties of Levy processes.
- The generator and the resolvent of a Levy process.
- Excessive functions
- Transience and Recurrence.

(3) Subordinators

- Definition and first properties.
- Passage across a level
- The Arc-Sine law.
- Regenerative sets and the ranges of subordinators.

(4) Local times and excursions of Markov processes

- Blumenthal's 0-1 Law.
- Regular points, holding points and instantaneous points of a Markov process.
- The construction of the local time of a Markov Process.
- Uniqueness of the local time at points.
- Inverse local time as a subordinator.
- Excursion measure and the excursion process.

(5) Fluctuation Theory of Levy Processes

- Maxima and Minima of Levy processes, ladder processes and reflected processes.
- Fluctuation Identities. The Wiener Hopf Factorization.

(6) The Markov property of local times of diffusion processes with fixed birth and death points.

- Necessary and sufficient conditions for the Markov property of the local time process.
- Bessel processes and the Ray Knight theorems.

(7) Continuity of the local time process.

- Dynkin's Isomorphism Theorem
- Necessary and sufficient conditions for the joint continuity of the local times of symmetric Markov processes.
- Necessary and sufficient conditions for the joint continuity of the local times of Levy processes.
- Sufficient conditions for the joint continuity of the local times of Markov processes.

(8) Permanent Processes associated with local times of Markov processes

- definition of Permanent Processes
- infinitely divisible Permanent Processes associated with Markov processes
- Isomorphism Theorems associated with Permanent Processes.

(9) Rosinski representation of infinitely divisible processes.

- 0-1 laws for infinitely divisible processes
- Rosinski representation of Permanent processes associated with Markov processes using local times.
- Open problem—are the sufficient conditions also necessary for the continuity of the local time of Markov processes.

(10) Stable Processes and the Scaling Property.

- Stable processes with index $0 < \alpha < 1$ and their sample path properties.
- Stable processes with index $1 \leq \alpha < 2$ and their sample path properties.
- Excursions of Stable Processes.