

098416 THEORY OF PROBABILITY

Lecturer	Dima Ioffe
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Time	Wednesday 15:30 - 18:30
Place	Room 153 IE Bloomfield
Language	English
Grading	Final take-home exam

Basic graduate-level course on the rigorous foundations of the probability theory.

1. Basic measure theory (ca. 3 weeks). σ -algebras, σ -additivity, measure and measure spaces, Carathéodory extension theorem, measurable functions, Lebesgue integral, monotone and dominated convergence theorems, Fatou's and Scheffé's lemmas, Radon-Nikodým theorem, Fubini's theorem.
2. Basic probability theory (ca. 3-4 weeks). Probability spaces, independence, Borel-Cantelli lemmas, Kolmogorov's 0 – 1 law, random variables, expectations, inequalities (Markov's, Chebyshev's, Jensen's, Cauchy-Schwarz, Hölder's, Minkowski's), modes of convergence, laws of large numbers, large deviation and concentration inequalities.
3. Martingales (ca. 3-4 weeks). Conditional expectations, super and sub-martingales, Doob's up-crossing lemma and convergence theorems, uniform integrability, \mathbb{L}_2 -martingales, applications.
4. Weak convergence (ca. 2-3 weeks). Weak convergence of probability measures, characteristic functions, central limit theorem, infinitely divisible distributions and stable laws.

REFERENCES

- [1] D. Williams, *Probability with Martingales*, Cambridge Univ. Press (1991).
- [2] R. Durrett, *Probability: Theory and Examples*, Wadsworth&Brooks, (1991).
- [3] S.R.S. Varadhan, *Probability Theory*, AMS, Courant Series 7, (2001).